Midterm Program Correctness,
March 6th 2015, 14:00-16:00h

- This midterm consists of three problems. You can earn 100 grade points: 15 points for problem 1, 25 points for problem 2, 50 points for problem 3, and 10 points for not misspelling your name and student number.

- Give complete annotations, and linear proofs. If you make too big steps in your proofs (even if they are correct), grade points will be subtracted.

- Use a pen. Do not use a pencil! Write neatly! If your handwriting needs guessing to read it, it is considered incorrect. Do not hand in scratch paper!

- This exam is a closed book exam. You are not allowed to use the reader, slides, notes, or any other material.

Problem 1 (15 pt). Given are the program variables $x, y : \mathbb{Z}$. Derive an annotated program fragment S that consists of exactly two assignments and satisfies

$$\{ 4 \cdot x + 2 \cdot y = 2 \cdot X + 4 \cdot Y \ \wedge \ 2 \cdot x - y = Y \}$$
$$S$$
$$\{ x = X \ \wedge \ y = Y \} .$$

Problem 2 (25 pt). Given is the program variable $n : \mathbb{Z}$. Derive an annotated conditional command T that satisfies

$$\{ X \geq 0 \ \wedge \ (n = 2 \cdot X - 3 \ \vee \ 4 \cdot X + n = 0) \}$$
$$T$$
$$\{ n = X \}$$

Problem 3 (50 pt). The function $f : \mathbb{N} \to \mathbb{N}$ is defined by the recurrence:

$$
\begin{aligned}
n = 0 \ &\Rightarrow \ f(n) = 1 \\
n > 0 \ \wedge \ n \bmod 2 = 0 \ &\Rightarrow \ f(n) = 1 + 2 \cdot f(n \text{ div } 2) \\
n > 0 \ \wedge \ n \bmod 2 = 1 \ &\Rightarrow \ f(n) = 2 \cdot f(n \text{ div } 2)
\end{aligned}
$$

Derive an annotated command U that satisfies the following specification:

$$\textbf{var } a, \ n : \mathbb{N};$$
$$\{ P : \ X = f(n) \}$$
$$U$$
$$\{ Q : \ X = a \}$$

Use a **while**-loop that maintains the invariant $J : \ X = a + b \cdot f(n) \ \wedge \ n \geq 0$. You will need active finalization.